Problems on Probability with solutions:

Example 1: A coin is thrown 3 times .what is the probability that atleast one head is obtained?

Sol: Sample space = [HHH, HHT, HTH, THH, TTH, THT, HTT, TTT]

Total number of ways = $2 \times 2 \times 2 = 8$. Fav. Cases = 7

P (A) = 7/8

OR

P (of getting at least one head) = 1 – P (no head)$\Rightarrow$ 1 – (1/8) = 7/8

Example 2: Find the probability of getting a numbered card when a card is drawn from the pack of 52 cards.

Sol: Total Cards = 52. Numbered Cards = (2, 3, 4, 5, 6, 7, 8, 9, 10) 9 from each suit $4 \times 9 = 36$

P (E) = 36/52 = 9/13

Example 3: There are 5 green 7 red balls. Two balls are selected one by one without replacement. Find the probability that first is green and second is red.

Sol: P (G) × P (R) = (5/12) x (7/11) = 35/132

Example 4: What is the probability of getting a sum of 7 when two dice are thrown?

Sol: Probability math - Total number of ways = $6 \times 6 = 36$ ways. Favorable cases = (1, 6) (6, 1) (2, 5) (5, 2) (3, 4) (4, 3) --- 6 ways. P (A) = 6/36 = 1/6

Example 5: 1 card is drawn at random from the pack of 52 cards.

(i) Find the Probability that it is an honor card.

(ii) It is a face card.

Sol: (i) honor cards = (A, J, Q, K) 4 cards from each suits = $4 \times 4 = 16$

P (honor card) = 16/52 = 4/13

(ii) face cards = (J,Q,K) 3 cards from each suit = $3 \times 4 = 12$ Cards.

P (face Card) = 12/52 = 3/13

Example 6: Two cards are drawn from the pack of 52 cards. Find the probability that both are diamonds or both are kings.

Sol: Total no. of ways = ${}^{52}C_2$

Case I: Both are diamonds = ${}^{13}C_2$

Case II: Both are kings = ${}^{4}C_{2}$

P (both are diamonds or both are kings) = $({}^{13}C_{2} + {}^{4}C_{2}) / {}^{52}C_{2}$

Example 7: Three dice are rolled together. What is the probability as getting at least one '4'?

Sol: Total number of ways = 6 × 6 × 6 = 216. Probability of getting number '4' at least one time = 1 – (Probability of getting no number 4) = 1 – (5/6) x (5/6) x (5/6) = 91/216

Example 9: Find the probability of getting two heads when five coins are tossed.

Sol: Number of ways of getting two heads = ${}^{5}C_{2}$ = 10. Total Number of ways = 2^5 = 32

P (two heads) = 10/32 = 5/16

Example 10: What is the probability of getting a sum of 22 or more when four dice are thrown?

Sol: Total number of ways = 6^4 = 1296. Number of ways of getting a sum 22 are 6,6,6,4 = 4! / 3! = 4

6,6,5,5 = 4! / 2!2! = 6. Number of ways of getting a sum 23 is 6,6,6,5 = 4! / 3! = 4.

Number of ways of getting a sum 24 is 6,6,6,6 = 1.

Fav. Number of cases = 4 + 6 + 4 + 1 = 15 ways. P (getting a sum of 22 or more) = 15/1296 = 5/432

Example 11: Two dice are thrown together. What is the probability that the number obtained on one of the dice is multiple of number obtained on the other dice?

Sol: Total number of cases = 6^2 = 36

Since the number on a die should be multiple of the other, the possibilities are

(1, 1) (2, 2) (3, 3) ------ (6, 6) --- 6 ways

(2, 1) (1, 2) (1, 4) (4, 1) (1, 3) (3, 1) (1, 5) (5, 1) (6, 1) (1, 6) --- 10 ways

(2, 4) (4, 2) (2, 6) (6, 2) (3, 6) (6, 3) -- 6 ways

Favorable cases are = 6 + 10 + 6 = 22. So, P (A) = 22/36 = 11/18

Example 12: From a pack of cards, three cards are drawn at random. Find the probability that each card is from different suit.

Sol: Total number of cases = ${}^{52}C_{3}$

One card each should be selected from a different suit. The three suits can be chosen in ${}^{4}C_{3}$ was

The cards can be selected in a total of $({}^{4}C_{3}) \times ({}^{13}C_{1}) \times ({}^{13}C_{1}) \times ({}^{13}C_{1})$

Probability = ${}^{4}C_{3} \times ({}^{13}C_{1})^3 / {}^{52}C_{3}$

= $4 \times (13)^3 / {}^{52}C_{3}$

Example 13: Find the probability that a leap year has 52 Sundays.

Sol: A leap year can have 52 Sundays or 53 Sundays. In a leap year, there are 366 days out of which there are 52 complete weeks & remaining 2 days. Now, these two days can be (Sat, Sun) (Sun, Mon) (Mon, Tue) (Tue, Wed) (Wed, Thur) (Thur, Friday) (Friday, Sat).

So there are total 7 cases out of which (Sat, Sun) (Sun, Mon) are two favorable cases. So, P (53 Sundays) = 2 / 7

Now, P(52 Sundays) + P(53 Sundays) = 1

So, P (52 Sundays) = 1 - P(53 Sundays) = 1 – (2/7) = (5/7)

Addition Rule 1: When two events, A and B, are mutually exclusive, the probability that A or B will occur is the sum of the probability of each event.

P(A or B) = P(A) + P(B)

Let's use this addition rule to find the probability for Experiment 1.

Experiment 1: A single 6-sided die is rolled. What is the probability of rolling a 2 or a 5?

Probabilities:

$$P(2) = \frac{1}{6}$$

$$P(5) = \frac{1}{6}$$

$$P(2 \text{ or } 5) = P(2) + P(5) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

Experiment 2: A spinner has 4 equal sectors colored yellow, blue, green, and red. What is the probability of landing on red or blue after spinning this spinner?

Probabilities:

$$P(\text{red}) = \frac{1}{4}$$

$$P(\text{blue}) = \frac{1}{4}$$

$$P(\text{red or blue}) = P(\text{red}) + P(\text{blue})$$

$$= \frac{1}{4} + \frac{1}{4}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

Experiment 3: A glass jar contains 1 red, 3 green, 2 blue, and 4 yellow marbles. If a single marble is chosen at random from the jar, what is the probability that it is yellow or green?

Probabilities:

$$P(\text{yellow}) = \frac{4}{10}$$

$$P(\text{green}) = \frac{3}{10}$$

$$P(\text{yellow or green}) = P(\text{yellow}) + P(\text{green})$$

$$= \frac{4}{10} + \frac{3}{10}$$

$$= \frac{7}{10}$$

In each of the three experiments above, the events are mutually exclusive. Let's look at some experiments in which the events are non-mutually exclusive.

Experiment 4: A single card is chosen at random from a standard deck of 52 playing cards. What is the probability of choosing a king or a club?

Probabilities:

$$P(\text{king or club}) = P(\text{king}) + P(\text{club}) - P(\text{king of clubs})$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

$$= \frac{4}{13}$$

In Experiment 4, the events are non-mutually exclusive. The addition causes the king of clubs to be counted twice, so its probability must be subtracted. When two events are non-mutually exclusive, a different addition rule must be used.

Additional Rule 2: When two events, A and B, are non-mutually exclusive, the probability that A or B will occur is:

P(A or B) = P(A) + P(B) - P(A and B)

In the rule above, P(A and B) refers to the overlap of the two events. Let's apply this rule to some other experiments.

Experiment 5: In a math class of 30 students, 17 are boys and 13 are girls. On a unit test, 4 boys and 5 girls made an A grade. If a student is chosen at random from the class, what is the probability of choosing a girl or an A student?

Probabilities: P(girl or A) = P(girl) + P(A) - P(girl and A)

$$= \frac{13}{30} + \frac{9}{30} - \frac{5}{30}$$

$$= \frac{17}{30}$$

Summary: To find the probability of event A or B, we must first determine whether the events are mutually exclusive or non-mutually exclusive. Then we can apply the appropriate Addition Rule:

Addition Rule 1: When two events, A and B, are mutually exclusive, the probability that A or B will occur is the sum of the probability of each event.

P(A or B) = P(A) + P(B)

Addition Rule 2: When two events, A and B, are non-mutually exclusive, there is some overlap between these events. The probability that A or B will occur is the sum of the probability of each event, minus the probability of the overlap.

P(A or B) = P(A) + P(B) - P(A and B)

Multiplication Rule 1: When two events, A and B, are independent, the probability of both occurring is:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

(Note: Another multiplication rule will be introduced in the next lesson.) Now we can apply this rule to find the probability for Experiment 1.

Experiment 1: A dresser drawer contains one pair of socks with each of the following colors: blue, brown, red, white and black. Each pair is folded together in a matching set. You reach into the sock drawer and choose a pair of socks without looking. You replace this pair and then choose another pair of socks. What is the probability that you will choose the red pair of socks both times?

Probabilities:

$$P(\text{red}) = \frac{1}{5}$$

$$P(\text{red and red}) = P(\text{red}) \cdot P(\text{red})$$

$$= \frac{1}{5} \cdot \frac{1}{5}$$

$$= \frac{1}{25}$$

Experiment 2: A coin is tossed and a single 6-sided die is rolled. Find the probability of landing on the head side of the coin and rolling a 3 on the die.

Probabilities:

$$P(\text{head}) = \frac{1}{2}$$

$$P(3) = \frac{1}{6}$$

$$P(\text{head and } 3) = P(\text{head}) \cdot P(3)$$

$$= \frac{1}{2} \cdot \frac{1}{6}$$

$$= \frac{1}{12}$$

Experiment 3: A card is chosen at random from a deck of 52 cards. It is then replaced and a second card is chosen. What is the probability of choosing a jack and then an eight?

Probabilities:

P(jack) $= \frac{4}{52}$

P(8) $= \frac{4}{52}$

P(jack and 8) = P(jack) · P(8)

$= \frac{4}{52} \cdot \frac{4}{52}$

$= \frac{16}{2704}$

$= \frac{1}{169}$

Experiment 4: A jar contains 3 red, 5 green, 2 blue and 6 yellow marbles. A marble is chosen at random from the jar. After replacing it, a second marble is chosen. What is the probability of choosing a green and then a yellow marble?

Probabilities:

P(green) $= \frac{5}{16}$

P(yellow) $= \frac{6}{16}$

P(green and yellow) = P(green) · P(yellow)

$= \frac{5}{16} \cdot \frac{6}{16}$

$= \frac{30}{256}$

$= \frac{15}{128}$

Each of the experiments above involved two independent events that occurred in sequence. In some cases, there was replacement of the first item before choosing the second item; this replacement was needed in order to make the two events independent. Multiplication Rule 1 can be extended to work for three or more independent events that occur in sequence. This is demonstrated in Experiment 5 below.

Experiment 5: A school survey found that 9 out of 10 students like pizza. If three students are chosen at random with replacement, what is the probability that all three students like pizza?

Probabilities:

P(student 1 likes pizza) $= \frac{9}{10}$

P(student 2 likes pizza) $= \frac{9}{10}$

P(student 3 likes pizza) $= \frac{9}{10}$

P(student 1 and student 2 and student 3 like pizza) $= \frac{9}{10} \cdot \frac{9}{10} \cdot \frac{9}{10} = \frac{729}{1000}$

All of the experiments above involved independent events with a small population (e.g. A 6-sided die, a 2-sided coin, a deck of 52 cards). When a small number of items are selected from a large population *without replacement*, the probability of each event changes so slightly that the amount of change is negligible. This is illustrated in the following problem.

Problem: A nationwide survey found that 72% of people in the United States like pizza. If 3 people are selected at random, what is the probability that all three like pizza?

Solution: Let L represent the event of randomly choosing a person who likes pizza from the U.S.

$P(L) \cdot P(L) \cdot P(L) = (0.72)(0.72)(0.72) = 0.37 = 37\%$

In the next lesson, we will address how to handle non-replacement in a small population.

Experiment 2: Mr. Parietti needs two students to help him with a science demonstration for his class of 18 girls and 12 boys. He randomly chooses one student who comes to the front of the room. He then chooses a second student from those still seated. What is the probability that both students chosen are girls?

Probabilities P(Girl 1 and Girl 2) = P(Girl 1) and P(Girl 2|Girl 1)

$$= \frac{18}{30} \cdot \frac{17}{29}$$

$$= \frac{306}{870}$$

$$= \frac{51}{145}$$

Experiment 4: Four cards are chosen at random from a deck of 52 cards without replacement. What is the probability of choosing a ten, a nine, an eight and a seven in order?

Probabilities: P(10 and 9 and 8 and 7) =

$$\frac{4}{52} \cdot \frac{4}{51} \cdot \frac{4}{50} \cdot \frac{4}{49} = \frac{256}{6{,}497{,}400} = \frac{32}{812{,}175}$$

Experiment 5: Three cards are chosen at random from a deck of 52 cards without replacement. What is the probability of choosing 3 aces?

Probabilities: P(3 aces) =

$$\frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} = \frac{24}{132{,}600} = \frac{1}{5{,}525}$$

Problem: A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?

Solution:

$$P(\text{Second}|\text{First}) = \frac{P(\text{First and Second})}{P(\text{First})} = \frac{0.25}{0.42} = 0.60 = 60\%$$

Let's look at some other problems in which we are asked to find a conditional probability.

Example 1: A jar contains black and white marbles. Two marbles are chosen without replacement. The probability of selecting a black marble and then a white marble is 0.34, and the probability of selecting a black marble on the first draw is 0.47. What is the probability of selecting a white marble on the second draw, given that the first marble drawn was black?

Solution:

$$P(\text{White}|\text{Black}) = \frac{P(\text{Black and White})}{P(\text{Black})} = \frac{0.34}{0.47} = 0.72 = 72\%$$

Example 2: The probability that it is Friday and that a student is absent is 0.03. Since there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent given that today is Friday?

Solution:

$$P(\text{Absent}|\text{Friday}) = \frac{P(\text{Friday and Absent})}{P(\text{Friday})} = \frac{0.03}{0.2} = 0.15 = 15\%$$

Summary: The conditional probability of an event B in relationship to an event A is the probability that event B occurs given that event A has already occurred. The notation for conditional probability is P(B|A), read as *the probability of B given A*. The formula for conditional probability is:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

The Venn Diagram below illustrates P(A), P(B), and P(A and B). What two sections would have to be divided to find P(B|A)?